ドーマー条件の見直し
財政安定化のための方策

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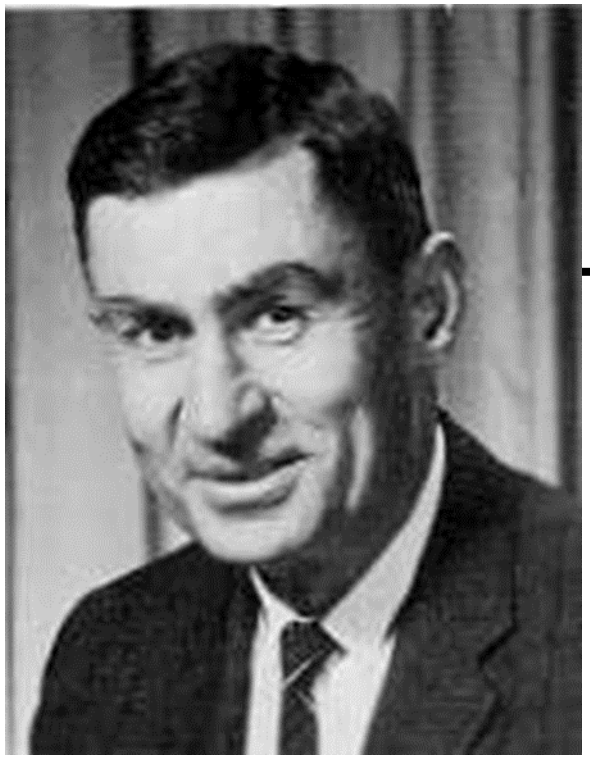
財政赤字のGDP比率(IMF)

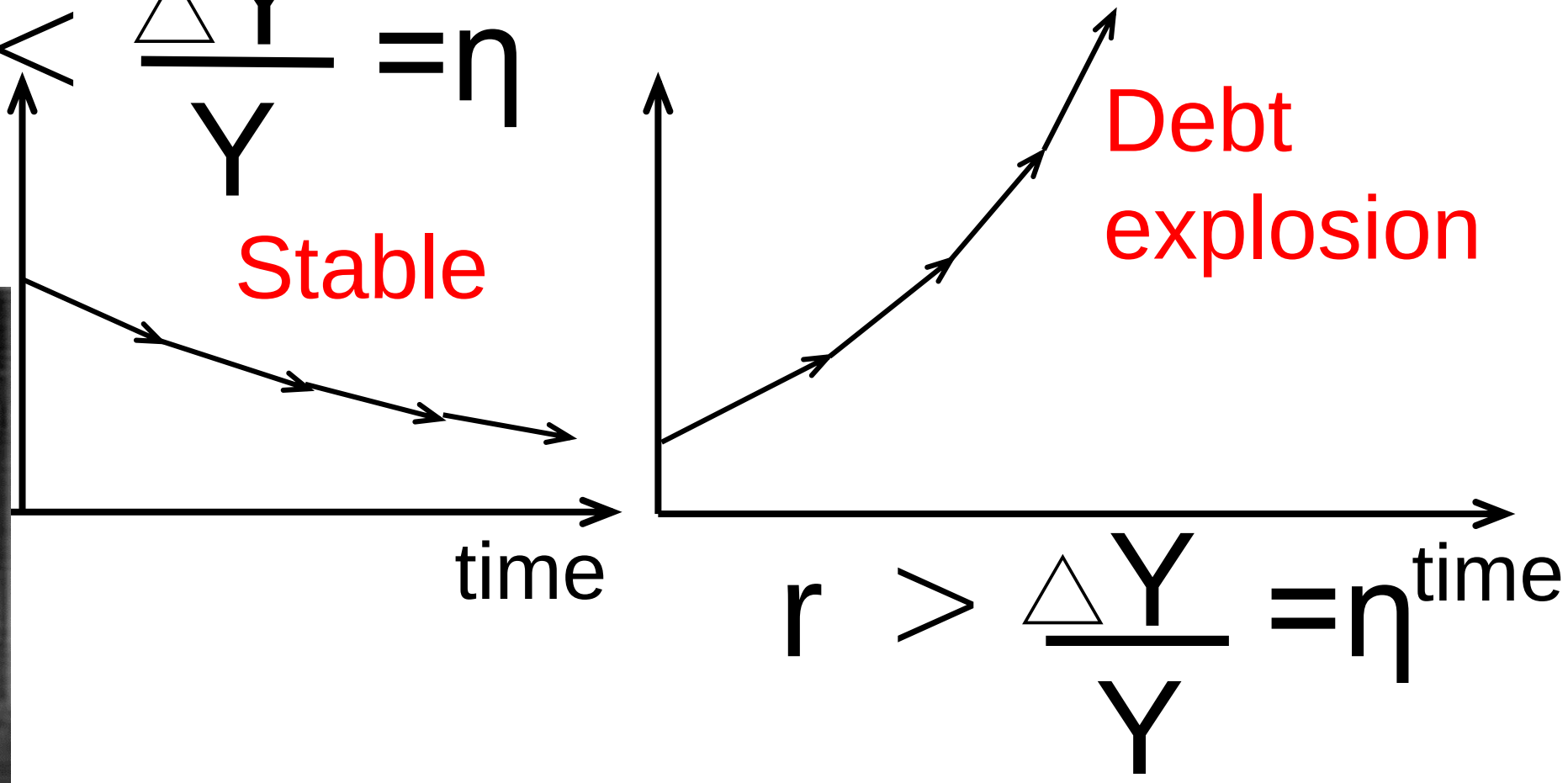
Debt to GDP Ratio (IMF Statistics) 2020

1. Japan (National Debt: ¥1,028 trillion ($9.087 trillion USD))
2. Greece (National Debt: €332.6 billion ($379 billion US))
3. Portugal (National Debt: €232 billion ($264 billion US))
4. Italy (National Debt: €2.17 trillion ($2.48 trillion US))
5. Bhutan (National Debt: $2.33 billion (USD))
6. Cyprus (National Debt: €18.95 billion ($21.64 billion USD))
7. Belgium (National Debt: €399.5 billion ($456.18 billion USD))
8. United States of America (National Debt: $19.23 trillion)
9. Spain (National Debt: €1.09 trillion ($1.24 USD))
10. Singapore (National Debt: $350 billion ($254 billion US))

ドマー条件：財政が安定するか発散するかの条件

$$r < \frac{\triangle Y}{Y} = \eta$$





$$r > \frac{\triangle Y}{Y} = \eta^{time}$$

r-g=Interest rate –Growth Rate(ΔY/Y)

Domar Condition of Fiscal Stability

The Domar condition is often used to judge whether the budget deficit is sustainable. The Domar condition is obtained from the government budget constraint:

$$G_t + r_t^B B_{t-1} = \Delta B_t + T_t,$$

where G_{t} is government spending, B_{t} is the stock of public debt, T_{t} is total tax revenues, and r_{t} is the interest rate for public debt. By dividing (1) by GDP Y_{t}, we can obtain

$$b_t - b_{t-1} = g_t - t_t + \frac{r_t - \eta_t}{1 + \eta_t} b_{t-1},$$

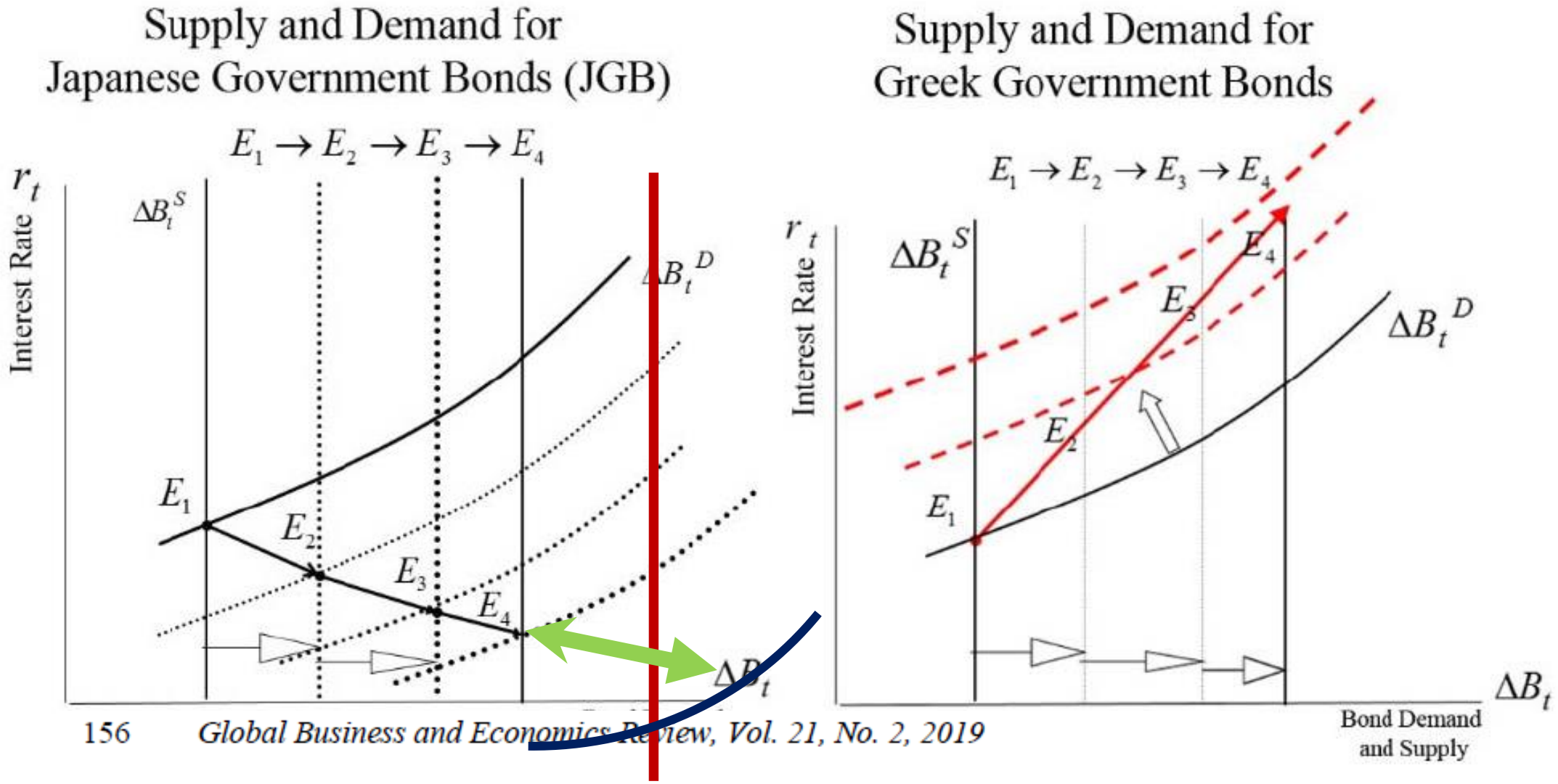
Interest Rate (r_t) > growth rate of the economy(η) → Unstable
Interest Rate (r_t) < growth rate of the economy(η) → Stable

Revival of Domar Condition by Paul Krugman and World Bank

1. Domar, E.D. (1944), “The Burden of Debt and the National Income”, American Economic Review, 34(4), pp. 798-827.
2. Krugman, P. (2020), “The case for permanent stimulus”, Mitigating the COVID Economic Crisis: Act Fast and Do Whatever It Takes, Edited by Richard Baldwinand Beatrice Weder di Mauro, A CEPR Press VoxEU.org eBook.
3, World Bank, East Asia and Pacific Economic Outlook , April 2021

ドーマー条件は、国債の供給式から導出されており、国債の需要については、言及していない。アメリカは基軸通貨国であり、危機が来るとドル需要（アメリカ国債）が増えるため、需要はいくらでも世界から来る。

Figure 1 Government bond markets of Japan and Greece (see online version for colours)



Optimal fiscal policy rule for achieving fiscal sustainability: *the Japanese case* Yoshino-Mizoguchi-Hesary (2018)

Table 1 Holders of Japanese and Greek Government bonds (2012)

Holders of Japanese Government bonds	*% of total*	*Holders of Greek Government bonds*	*% of total*
Bank and postal savings	45	Overseas investors	33
Life and non-life insurance	20	Domestic investors	21
Public pension funds	10	European Central Bank	18
Private pension funds	4	Bilateral loans	14
Bank of Japan	8	Social pension funds	6
Overseas investors	5	International Monetary Fund	5
Households	5	Greek domestic funds	3
Others	3		

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Optimal fiscal policy rule for achieving fiscal sustainability: *the Japanese case* **Yoshino-Mizoguchi-Hesary (2019)**

10-Year Government Bonds Yields

(Source) Bloomberg

Derivation of government bond demand

$$U(r_t, \sigma_t) = r_t - \beta\sigma_t^2. \quad (4)$$

$$r_t = \alpha r_t^B + (1-\alpha) r_t^I. \quad (5)$$

$$\sigma_t^2 = \alpha^2(\sigma_t^B)^2 + (1-\alpha)^2(\sigma_t^I)^2 + 2\alpha(1-\alpha)\sigma_t^{BI}, \quad (6)$$

where σ^B shows the risk of investing into government bonds.

国債需要の導出

$$\alpha^* = \frac{\frac{1}{\beta}(r_t^B - r_t^I) + (\sigma_t^I)^2 + 2\sigma_t^{BI}}{2(\sigma_t^B)^2 + 2(\sigma_t^I)^2 - 4\sigma_t^{BI}}. \quad (7)$$

$$\Delta B_t^d = b_0 + b_1(\sigma_t^B, \sigma_t^I)(r_t^B - r_t^I). \quad (8)$$

Likewise the demand by foreign investors in government bonds

$$\Delta B_t^f = f_0 + f_1(\sigma_t^B, \sigma_t^f)\left[r_t^B - \left\{r_t^f + \frac{(e_t^e - e_t)}{e_t}\right\}\right], \quad (9)$$

国内投資家による需要

Demand for government bonds by domestic investors and foreign investors

$$\Delta B_t^D = (b_0 + f_0) + f_1(\sigma_t^B, \sigma_t^f)\left[r_t^B - \left\{r_t^f + \frac{(e_t^e - e_t)}{e_t}\right\}\right] + b_1(\sigma_t^B, \sigma_t^I)(r_t^B - r_t^I) \quad (10)$$

海外

Simultaneous equation system of supply and demand for government bonds

$$\Delta B_t^D = (b_0 + f_0) + f_1(\sigma_t^B, \sigma_t^f)\left[r_t^B - \left\{r_t^f + \frac{(e_t^e - e_t)}{e_t}\right\}\right] + b_1(\sigma_t^B, \sigma_t^I)(r_t^B - r_t^I) \quad (10)$$

$$\Delta B_t = (G_t - T_t) + r_t^B \times B_{t-1} - \Delta M_t. \quad (12) \qquad \text{Supply}$$

From equation (10) and equation (12), the equilibrium interest rate on the government bond will be obtained as follow:

$$r_t^{B^*} = \frac{(G_t - T_t) - \Delta M_t - (b_0 + f_0) + f_1\left(r_t^f + \frac{e_t^e - e_t}{e_t}\right) + b_1 r_t^I}{(b_1 + f_1) - B_{t-1}}. \quad (13)$$

(i). An increase in government spending will increase government bond issuance,

(ii). The Bank of Japan uses open market operations to increase the money supply

(iii). An increase in demand for government bonds by banks, insurance companies,

(iv). An increase in the outstanding amount of government bonds

(v). If a foreigner sells Greece government bonds and shifts to overseas government bonds,

Fiscal stabilization conditions that replace the Domar conditions derived from the simultaneous equations of government bond demand and government bond supply

$$\Delta B_t = (G_t - T_t) + r_t^{B^*} \times B_{t-1} - \Delta M_t. \qquad (14)$$

$$\frac{\partial \Delta B_t}{\partial B_{t-1}} = \frac{\partial r_t^{B^*}}{\partial B_{t-1}} B_{t-1} + r_t^{B^*}, \quad (15)$$

where

$$\frac{\partial r_t^{B^*}}{\partial B_{t-1}} = \frac{(G_t - T_t) - \Delta M_t - (b_0 + f_0) + f_1\left(r_t^f + \frac{e_t^e - e_t}{e_t}\right) + b_1 r_t^I}{[(b_1 + f_1) - B_{t-1}]^2} = \frac{r_t^{B^*}}{\left[\left((b_1 + f_1) - B_{t-1}\right)\right]}.$$

Equation (15) can be rewritten as follows.

$$\frac{\partial \Delta B_t}{\partial B_{t-1}} = \left(\frac{1}{1 - \frac{B_{t-1}}{b_1 + f_1}}\right) r_t^{B^*} \quad (16)$$

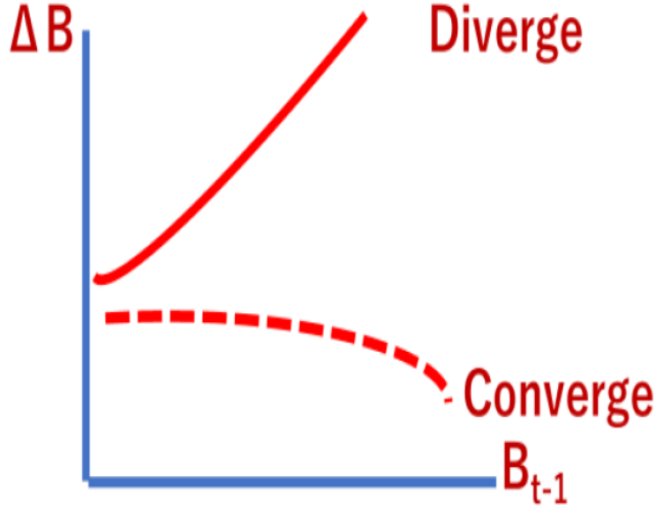
This implies

$$\frac{\partial \Delta B_t}{\partial B_{t-1}} \gtreqless 0 \Leftrightarrow 1 \gtreqless \frac{B_{t-1}}{b_1 + f_1}. \quad (17)$$

国債供給と国債需要の同時方程式から求められる財政安定化条件

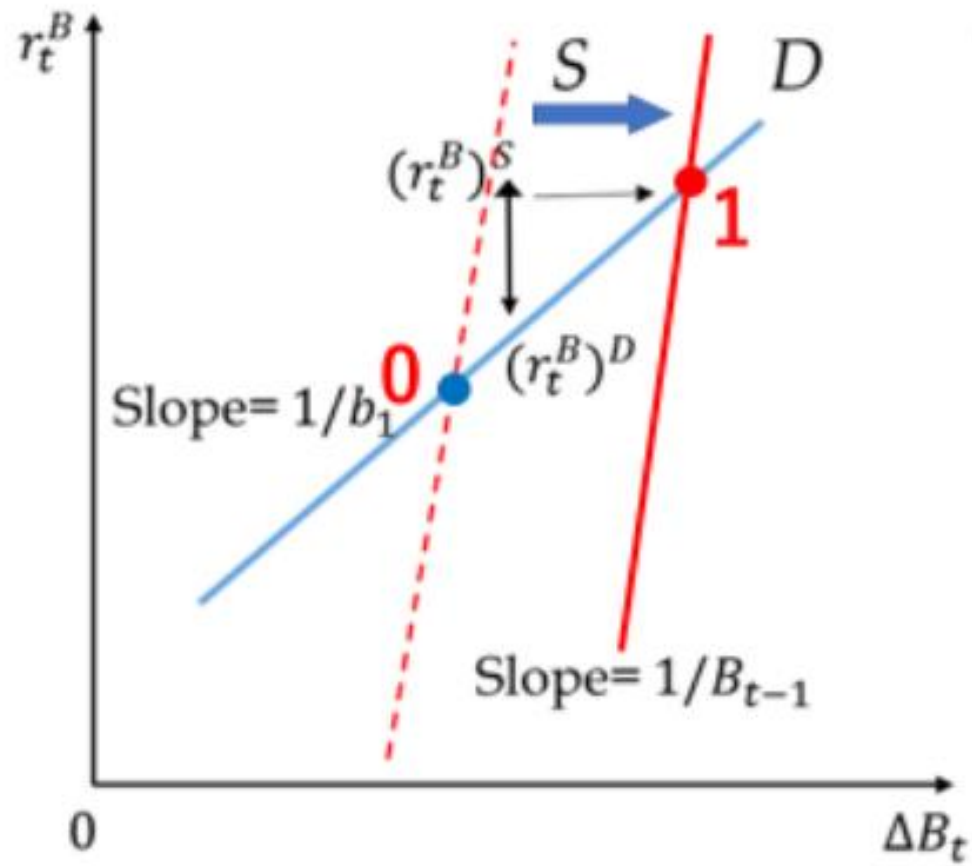
$$\frac{\delta \Delta B_t}{\delta B_{t-1}} = \left(\frac{1}{1 - \frac{B_{t-1}}{b_1 + f_1}} \right) r_t^{B^*} < 0 \quad (16) \quad \textit{Stability Condition}$$

Accumulated Government bond (B_{t-1})
$<$ Interest elasticity of Demand (b_1+f_1)

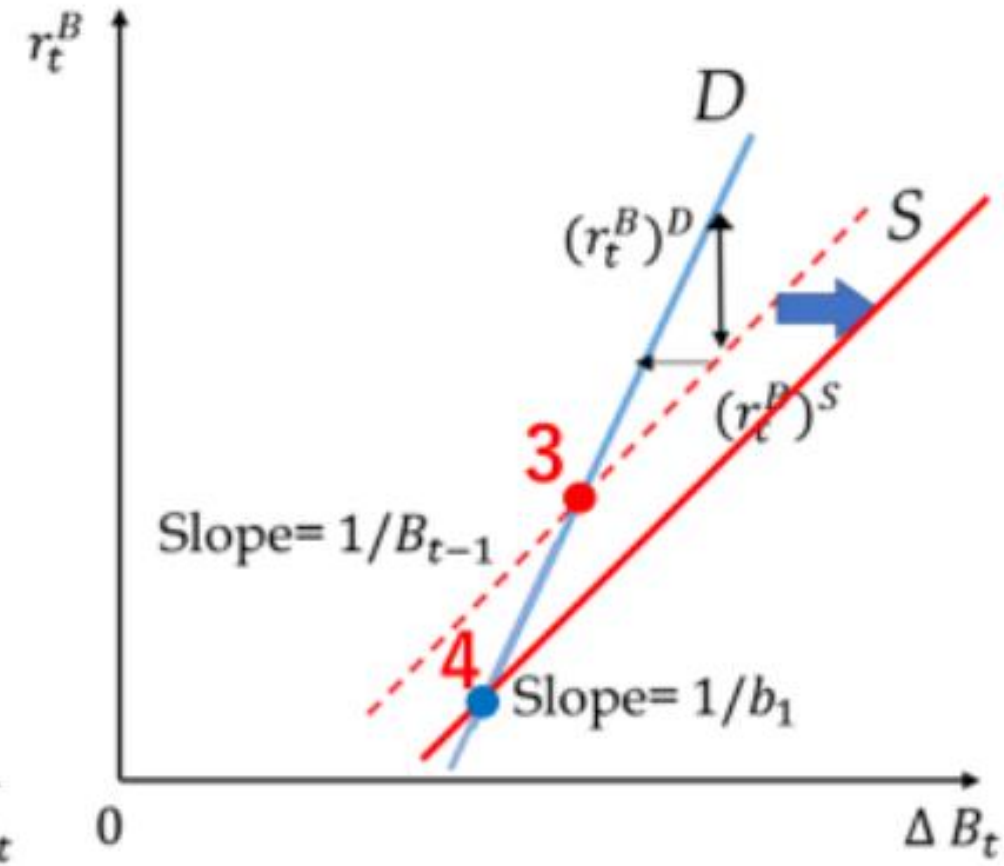


Revised Budget Stability Condition

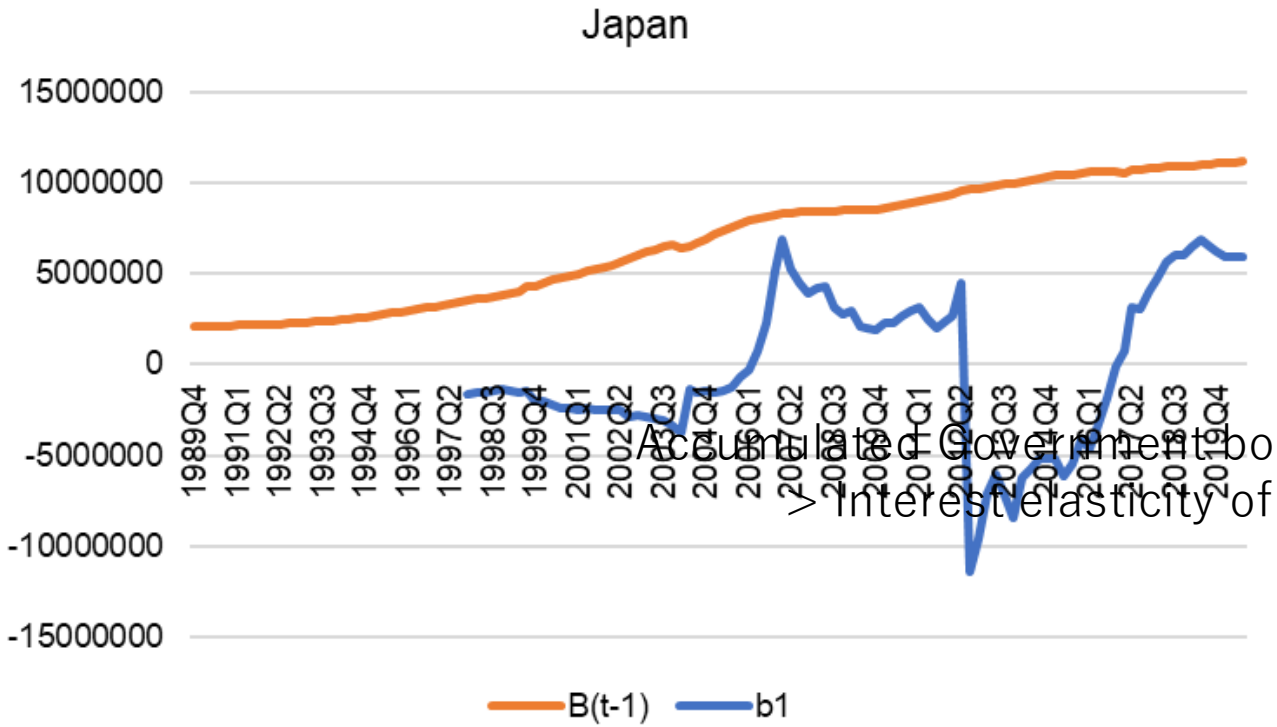
Explosion case ($B_{t-1} < b_1$)

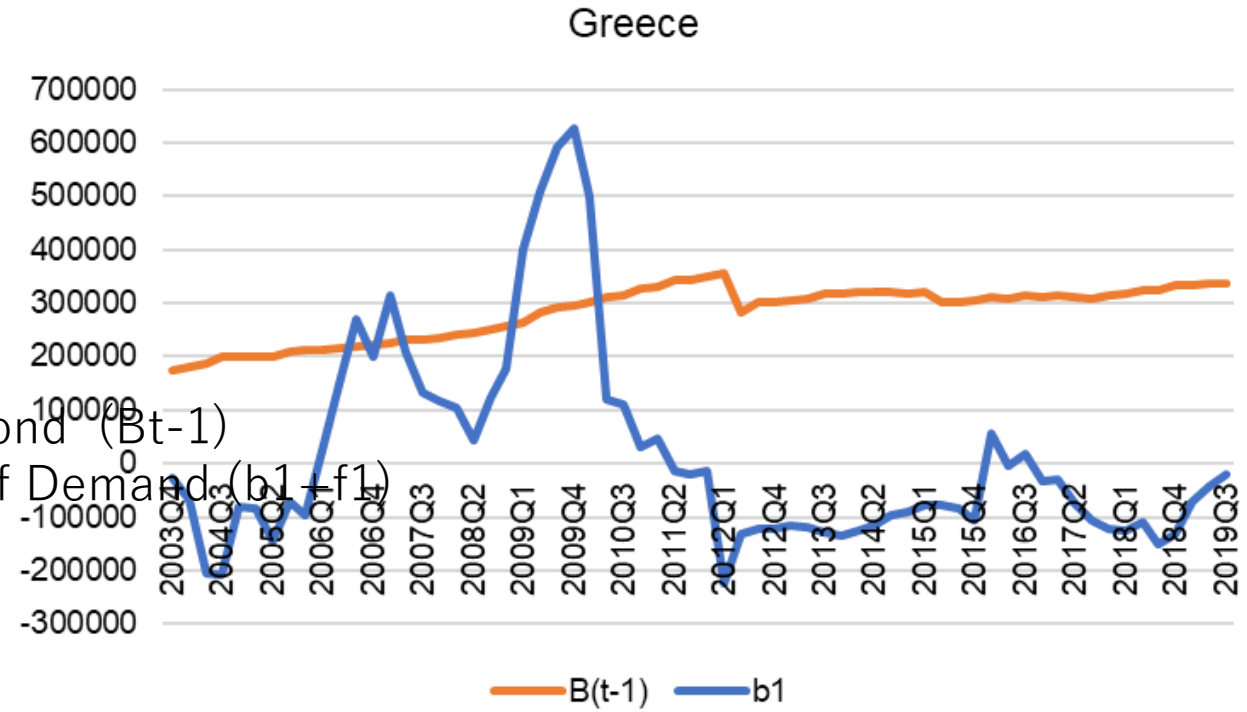


Stable case ($B_{t-1} > b_1$)



日本とギリシャの比較



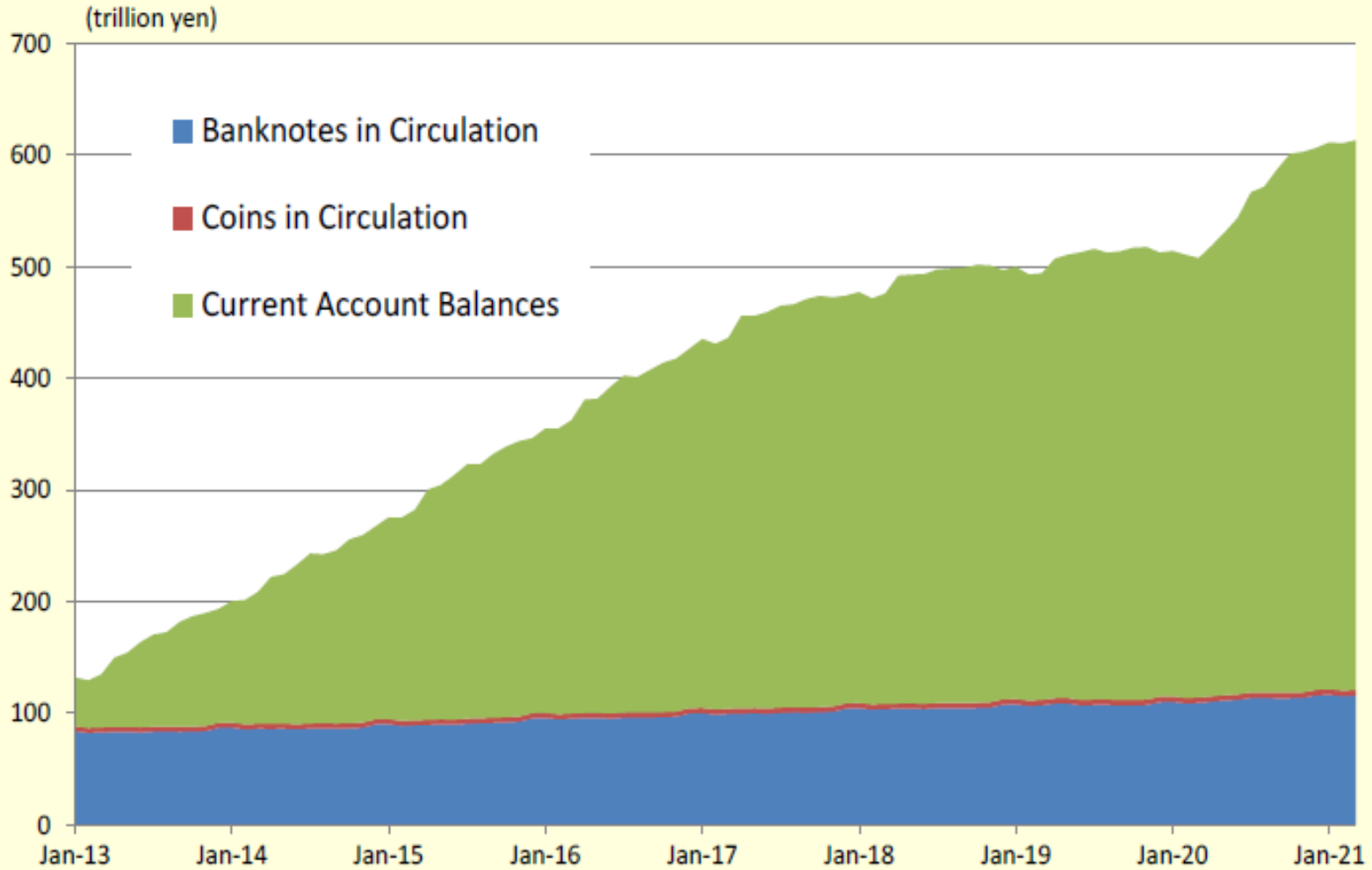


Accumulated Government bond (Bt-1)
＞Interest elasticity of Demand (b1+f1)

Global Solutions Journal (2020)

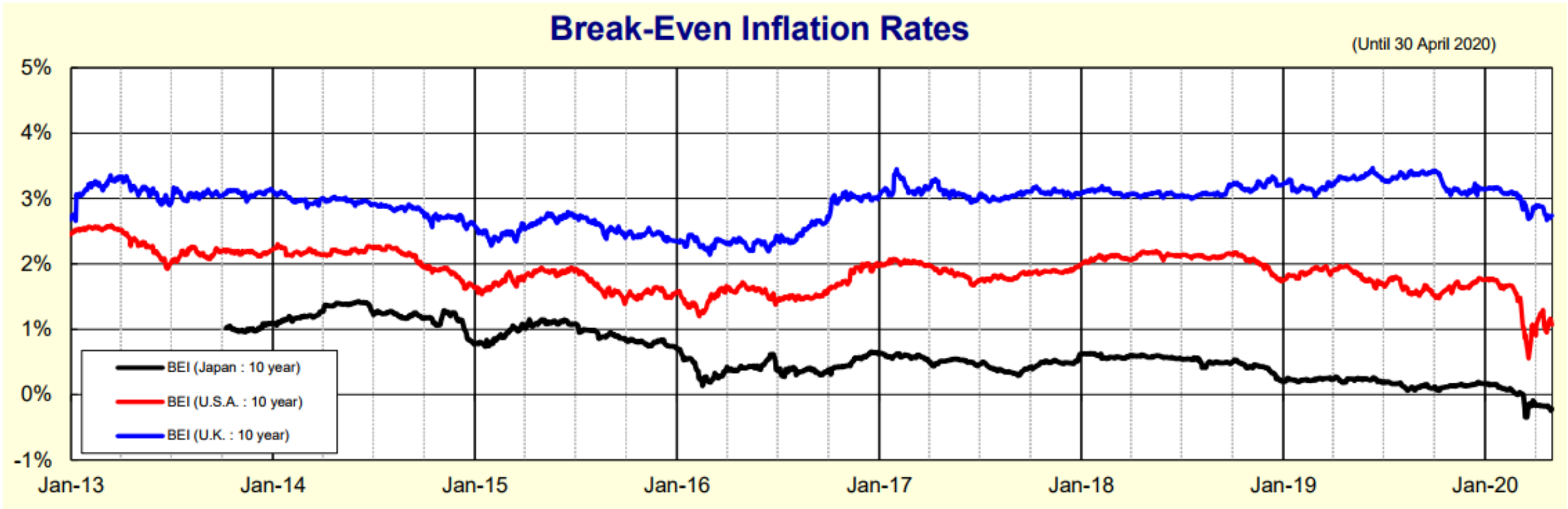
Accumulated Government bond (B_{t-1})
＜ Interest elasticity of Demand (b_1+f_1)

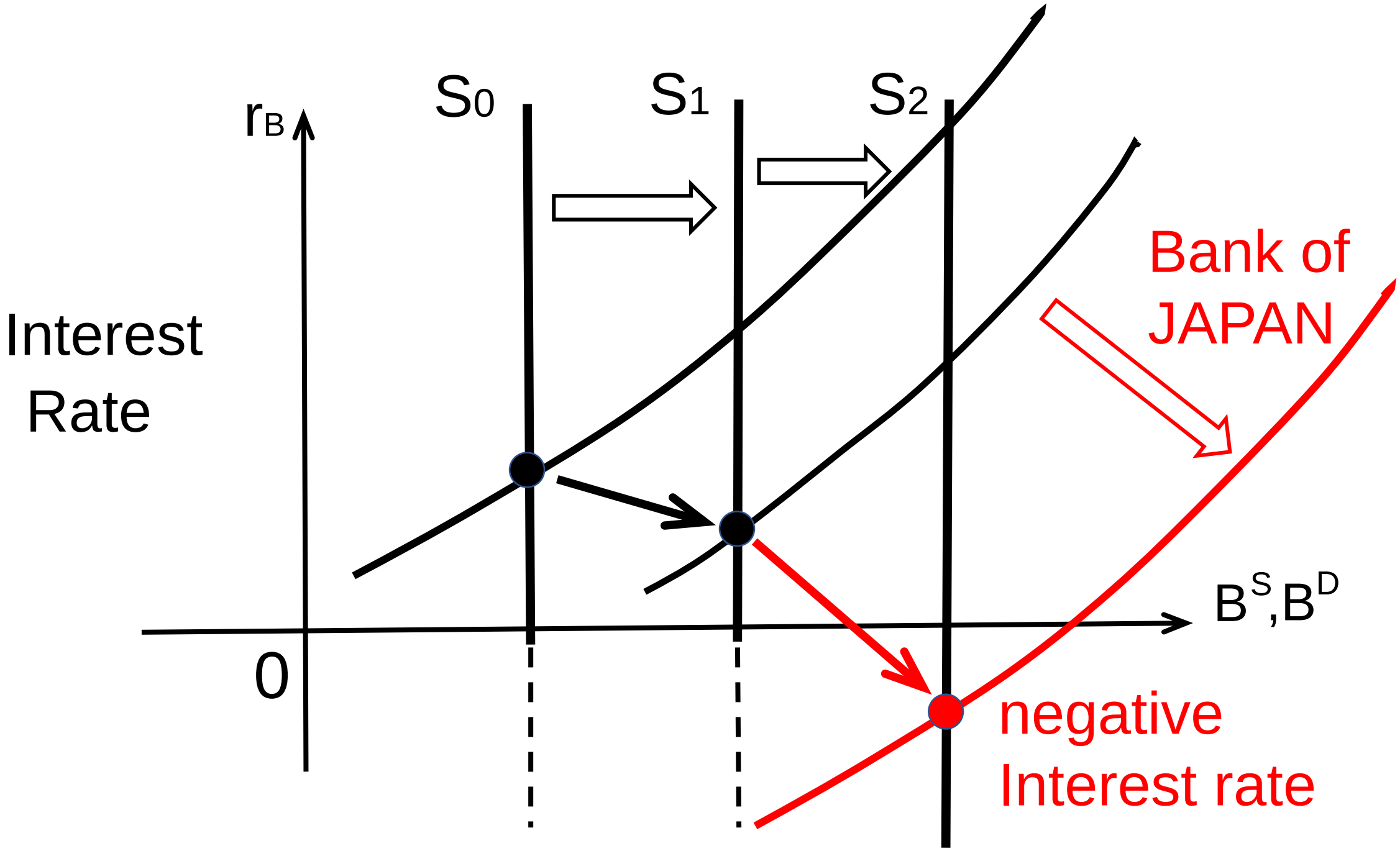
Monetary Base



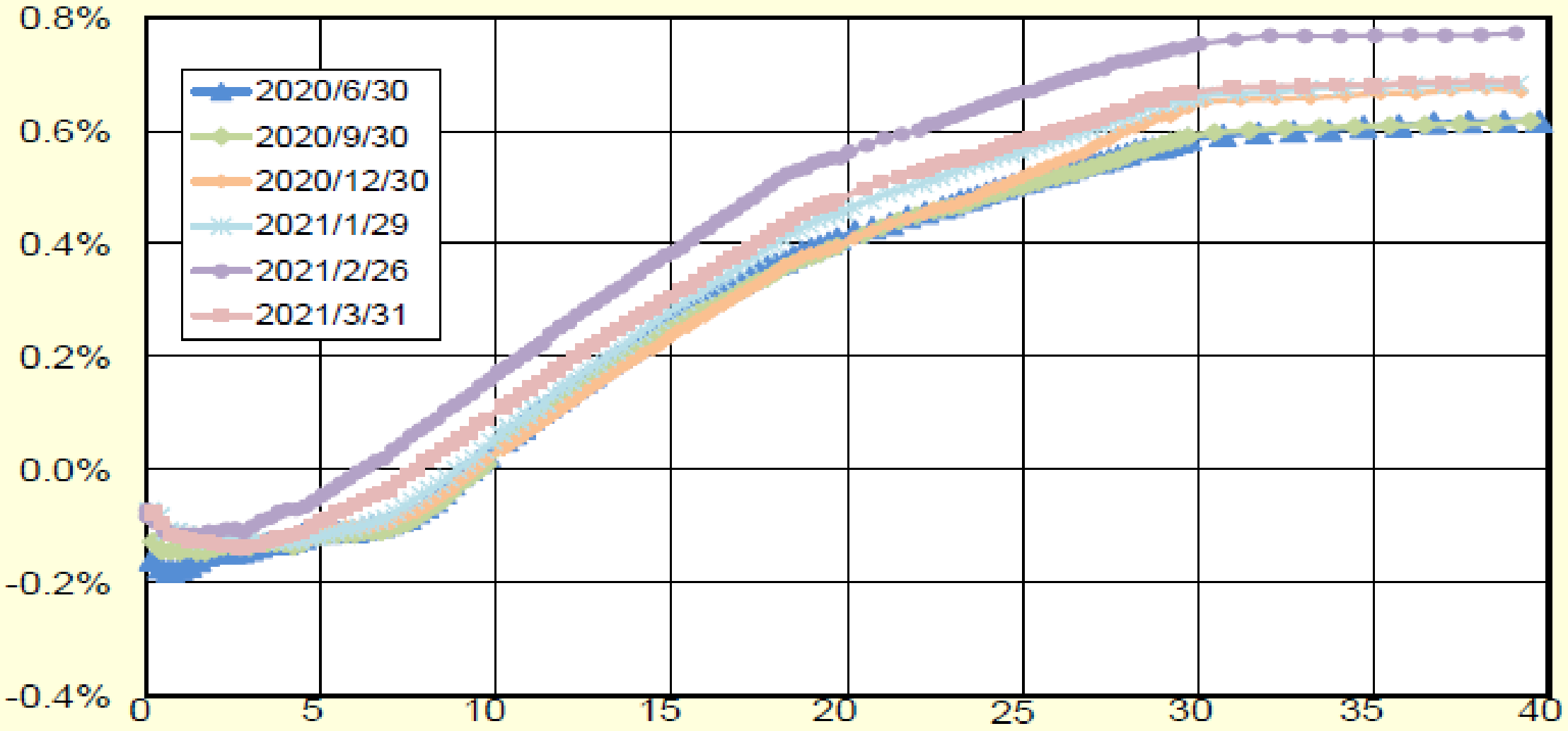
Source: Bank of Japan

	Apr 2013 (Actual)	Mar 2021 (Actual)
Monetary Base	**155**	**644**
JGBs	**98**	**496**
T-Bills	**36**	**36**
CP	**1.4**	**2.9**
Corporate Bonds	**2.9**	**7.5**
ETFs	**1.7**	**35.9**
J-REITs	**0.13**	**0.66**
Total Assets	**175**	**714**


Break-Even Inflation Rates
(Until 30 April 2020)
5%
4%
3%
2%
1%
0%
-1%
Jan-13
Jan-14
Jan-15
Jan-16
Jan-17
Jan-18
Jan-19
Jan-20
BEI (Japan : 10 year)
BEI (U.S.A. : 10 year)
BEI (U.K. : 10 year)

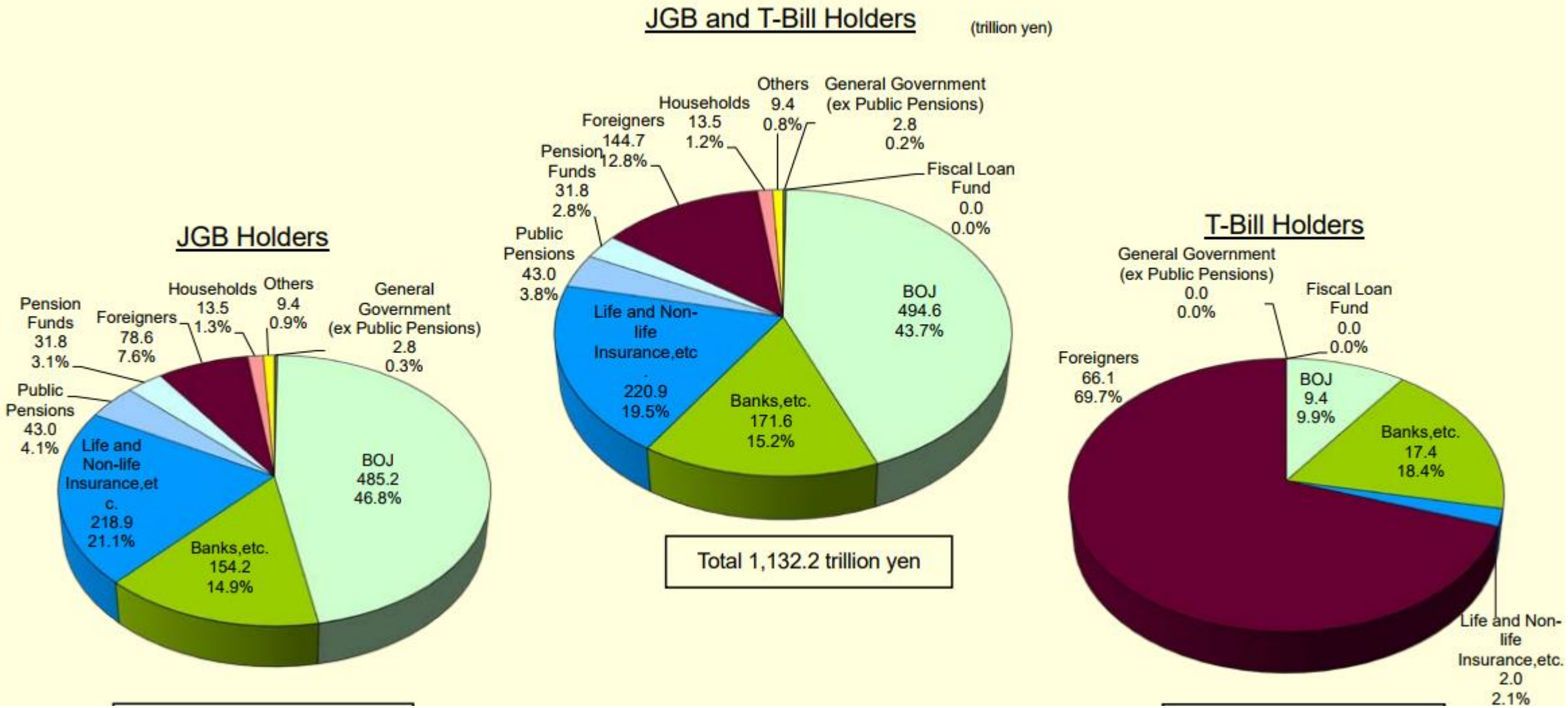

Interest
Rate
r_B
S_0
S_1
S_2
Bank of
JAPAN
B^S, B^D
0
negative
Interest rate

JGB Yield Curves



Source: Japan Bond Trading Co.,Ltd.

Breakdown by JGB and T-Bill Holders (The end of Dec. 2019 QE)



財政安定化のための条件

$$\Delta B_t = G_t - tY_t + r_t^B \times B_{t-1} - \Delta M_t$$

$$\frac{\partial \Delta B_t}{\partial B_{t-1}} = \frac{\partial G_t}{\partial B_{t-1}} - t \cdot \frac{\partial Y}{\partial B_{t-1}} + \left\{ \frac{\partial r_t^{B*}}{\partial B_{t-1}} \cdot B_{t-1} \right\} + r_t^{B*} - \frac{\partial \Delta M_t}{\partial B_{t-1}}$$

1，歳出削減
2，経済成長による税収増
3，金利が上昇しないようにする
4，金融政策に永遠には頼れない

なるべく高齢まで社会貢献し,給与は生産性に依存して決定

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Revisiting the public debt stability condition

Rethinking the Domar condition after COVID-19

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